ON THE ANALYSIS OF FACTORIAL EXPERIMENTS WITH TYPE I CENSORED DATA

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1. Introduction

Consider a factorial experiment with n replications per treatment combination in which the observations are subject to Type I (time) censoring. Under this type of censoring the observations are known exactly only if they are less than or equal to an *a priori* selected value T_0. All that is known about other observations is that they exceed T_0. Only censoring on the right is considered here, since the extension to censoring on the left is straightforward. The problem considered in this paper is how to estimate and test the parameters in a typical linear model for the mean observation at each treatment combination using Type I censored data. The models for the distribution of the dependent variable considered here are the exponential, Weibull, Type I extreme-value, normal, and lognormal distributions.

The above problem arose in the course of current research by West, *et al.* (1974) in social psychology investigating the phenomenon of helping behavior and what variables influence helping behavior. A 2^4 factorial experimental design was used to investigate the relationship between the dependent variable Y and four independent variables. Here Y=elapsed time until a passing motorist stops to help a confederate, i.e., an employee of the experimenter, standing at the roadside next to a car with a raised hood. The four two-level factors are the sex and race of the confederate, the racial composition of the neighborhood in which the experimental trial occurred, and the proximity of the trial to a college campus. There were eight trials per treatment combination. For economic reasons each trial was halted at 15 minutes if no help had been given. There was no replacement if help was given prior to 15 minutes. Note that in this problem, the Type I censoring occurs at T_0 = 900 seconds.

The similarity of this problem to problems encountered in life testing situations is apparent. In life testing situations the dependent variable is typically time until failue or death, whereas the dependent variable here is time until helping or time until success. Clearly, this poses no difficulty.

Section 2 summarizes results available in the statistical literature relating to parameter estimation and hypothesis testing for various experimental designs when the data are Type I, censored on the right, and when the model being considered is exponential, Weibull, Type I extreme-value, normal, or lognormal. Research relating to nonparametric approaches to this problem is not considered. Section 3 describes a general procedure utilizing linear models and maximum likelihood estimatiors (MLE's) for the analysis of factorial experiments with censored data. Section 4 illustrates the application of this procedure and Section 5 presents concluding remarks.

2. Literature review

After the author was first confronted by the social psychological problem given in Section 1, an extensive review of the life testing literature was undertaken. Though the literature on the analysis of Type I censored data contains much work on components of the problem considered here, a complete solution was not found. This section reviews results for problem components, and Section 3 presents a procedure which synthesizes these components into a complete solution to the problem. This section is organized as follows. Results are presented for five parametric models mentioned in Section 1. For each model results relating to the MLE's of the parameters in a one-sample problem are presented. This is a problem in which all observations are gathered under one set of conditions. Then results relating to best linear unbiased estimates (BLUE's) are given. Most of the work relating to the problem posed in Section 1 has been done for a one-sample problem. Progress on estimation and testing problems in one-, two-, and k-sample problems is presented. The review then considers any work done on testing and estimation relative to linear models for the parameters of the distributions.

Bibliographies on life testing and related results have been published by Mendenhall (1958) and Govindarajulu (1964). A new book which summarizes much work in this area is Mann, Schafer, and Singpurwalla (1974).

2.1. The exponential model

Let $t_1,t_2,\ldots,t_n$ denote observations made on a random variable T having the (negative) exponential distribution with cumulative distribution function

$$F_T(t) = 1 - \exp(-t/\theta),\ 0 \le t < \infty,\ 0< \theta < \infty,$$
$$= 0,\quad t < 0 .$$

If the data are Type I censored at time T_0, the exact values of the t_i are known only if $t_i \le T_0$. Define

$$x_i = t_i ,\ \text{if}\ \ t_i \le T_0 ,$$
$$= T_0 ,\ \text{if}\ \ t_i > T_0 ,\ \text{for}\ \ i = 1,2,\ldots,n,\ \text{and}$$

let r denote the number of $t_i \le T_0$. Using life testing terminology, T is the time to failure and r is the number of failures before the censoring time. The parameter θ is the mean of T and, in a life testing context, is referred to as the mean time between failures. Of course, the exponential density can also be parametrized using the failure rate $\lambda=1/\theta$. The failure rate is defined to be $f(t)/[1-F(t)]$, where f(t) is the probability density function of T.

Various authors, including Bartlett (1953), Bartholomew (1957), Deemer and Votaw (1955), and Littel (1952), have shown that the MLE of θ when the data are Type I censored is $\hat{\theta}= \sum_{i=1}^{n} x_i/r$.

Deemer and Votaw also showed that the asymptotic variance of $\hat{\theta}$ is

$$\theta^2[n(1 - \exp(-T_0/\theta)]^{-1} .$$

Since $\hat{\theta}$ is an MLE, and certain mild regularity conditions (Mann, *et al.*, 1974, p. 82) are satisfied here, the sampling distribution of $\hat{\theta}$ is asymptotically normal.

For small samples, Bartholomew (1957) found that $\hat{\theta}$ was biased and provided an exact expression for the bias. Mendenhall and Lehman (1960) gave tables to aid in computing the exact mean and variance of $\hat{\theta}$. Bartholomew (1963) derived the exact distribution of $\hat{\theta}$, which is too cumbersome unless n is very small. He suggested that n be greater than 40 for $\exp(-T_0/\theta)=0.10$ or n be greater than 80 for $\exp(-T_0/\theta) = 0.25$ before assuming that $\hat{\theta}$ is approximately normal. He derived another statistic, x in his notation, which is approximately normal for n as small as ten, where

$$x = r\left(\frac{\hat{\theta} - \theta}{\theta}\right)\{n[1 - \exp(-T_0/\theta)]\}^{-\frac{1}{2}} .$$

Note that Bartholomew's Equation (7) for x contains a typographical error. This statistic can be used to test hypotheses about θ or to construct confidence intervals for θ in one-sample problems.

For the exponential model, results for BLUE's comparable to those cited above for MLE's have not been published, to the best of the author's knowledge. This is not surprising since the MLE's can be expressed in closed form in this situation. Of course, it would be possible to construct BLUE's using results published for the Weibull distribution, since a two-parameter Weibull random variable with shape parameter equal to one is an exponential random variable.

No results were found relating to hypothesis testing or confidence interval estimation for the exponential parameter θ in two- or k-sample problems with Type I censoring. Mann, *et al.* (1974) give a test for θ in a two-sample problem, but their model assumes Type II censoring. Zelen (1959) discusses the analysis of a factorial experiment when the dependent variable is assumed to be exponentially distributed and the data are Type II censored. The data at each treatment combination are said to be Type II censored if the replications are observed until a preselected number r fail. At the time of the r'th failure for a particular treatment combination, all remaining unfailed replications are censored.

A number of authors have considered the problem of estimating the parameters of a linear model for θ in the exponential distribution. A typical situation is one in which the random variable is survival time in a medical study and the data analysis is to examine whether the survival time varies by some covariate, such as age. However, none of the work in this area provides a solution to the problem posed in Section 1 because either the articles do not consider censored data (Fiegl and Zelen, 1965; Glasser, 1967) or they do not consider linear models using Type I censored data (Cox, 1964; Sprott and Kalbfleisch, 1969; Cox, 1972; Prentice, 1973). Zippin and Armitage (1966) generalize the work of Fiegl and Zelen (1965) by considering censored data. The method of Zippin and Armitage could be used to solve the Section 1 problem in the case of one factor at two levels. For more complicated experimental designs, their method would be much more cumbersome and time-consuming than the method proposed in Section 3. For instance, one part of their method would require the iterative solution of k simultaneous non-linear equations in k unknowns, where k is the number of treatment combinations in the experimental design under consideration.

2.2. The Weibull and Type I extreme-value distributions

Let $t_1,t_2,\ldots,t_n$ denote observations on a random variable T having a two-parameter Weibull distribution with cumulative distribution function

$$F_T(t) = 1 - \exp[-(t/\alpha)^\beta], \quad 0\le x<\infty, \quad 0<\alpha, \quad \beta<\infty .$$

The parameter α is the scale parameter and β is the shape parameter of the Weibull distribution. If $\beta=1$, the distribution has a constant failure rate and T is exponential with parameter α. If $\beta<1$, the distribution has a decreasing failure rate, while if $\beta>1$, it has an increasing failure rate. It is well known that $Y=\ell n T$ has the Type I (smallest) extreme-value distribution with cumulative distribution function

$$F_Y(y) = 1 - \exp\{-\exp[(y - u)/b]\}, \quad -\infty < y < \infty ,$$

where $b=1/\beta$, $u=\ell n\alpha$. (All logarithms used in this paper are natural logarithms.) Thus, methods developed for one of these models can also be used for the other.

For Type I censored data, Cohen (1965) has developed a procedure for determining the MLE's of the parameters α and β. Let r denote the number of failures before the censoring time T_0. The first step is to obtain the MLE $\hat{\beta}$ by solving the following equation for $\hat{\beta}$:

$$\frac{\sum_{i=1}^{r} t_i^{\hat{\beta}}\ell n t_i + (n-r)T_0^{\hat{\beta}}\ell n T_0}{\sum_{i=1}^{r} t_i^{\hat{\beta}} + (n-r)T_0^{\hat{\beta}}} - \frac{1}{\hat{\beta}} - \frac{1}{r}\sum_{i=1}^{r}\ell n\, t_i = 0 .$$

Then the MLE $\hat{\alpha}$ is obtained by solving the following equation for $\hat{\alpha}$:

$$\hat{\alpha}^{\hat{\beta}} = \frac{1}{r}\sum_{i=1}^{r} t_i^{\hat{\beta}} + (n - r)T_0^{\hat{\beta}} .$$

Cohen also derived the asymptotic variance-covariance matrix of $(\hat{\alpha},\hat{\beta})$.

Methods for estimating the parameters of a linear model for a percentile of the distribution of T when the data are subject to Type II censoring have been presented by Lieblein and Zelen (1956). Their approach is similar to that of Nelson and Hahn (1972, 1973). Additional results relating to linear estimation are given by Mann, *et al.* (1974).

2.3. The normal and lognormal models

Let $t_1,t_2,\ldots,t_n$ denote n independent observations of the random variable T which is normally distributed with mean μ and variance σ^2. It is well known that, if $T=\ell n V$, then V is lognormally distributed. Thus, methods developed for one of these models can be used for the other. For Type I censored data, Cohen (1961) gives formulas and tables for evaluating the MLE's of μ and σ^2 and gives asymptotic formulas for the variances of the MLE's. The author is not aware of any published results relating to BLUE's for μ and σ^2 in the Type I censoring model.

Nelson and Hahn (1972, 1973) develop methods for estimating the parameters of a linear model for μ when the data are Type II censored. They consider methods which utilize BLUE's for μ at each setting of the independent variables and thereby ignore the information in the censored observations. They maintain that the distinction between Type I and Type II censoring can be ignored in practical situations. This contention and a comparison of their method to the method of Section 3 are topics of continuing research. Sampford and Taylor (1959) considered the analysis of Type II censored data in a randomized blocks experiment using a normal model.

3. A general procedure for testing and estimation in the analysis of factorial experiments with Type I censored data

This section describes a general procedure which can be used for the analysis of factorial experiments with Type I censored data in the many situations noted in Section 2 for which there currently is no analytic method available. The procedure will be described using a two-factor experiment. Extensions to experiments containing more factors are obvious.

Consider a factorial experiment in which factor A at a levels is crossed with factor B at b levels with n replications at each treatment combination. Label the treatment combinations (i,j), i=1,2,...,a; j=1,2,...,b. The data consist of abn observations on the dependent variable T. There is Type I censoring at the value T_0 at each treatment combination. Assume that interest centers on θ_{ij}, a parameter of the distribution of T in treatment combination (i,j), and how θ_{ij} varies across the treatment combinations. This section develops a procedure for investigating the θ_{ij}'s without making any restrictive assumptions about the specific form of the distribution of T.

To investigate the relationship between θ_{ij} and the factors A and B, consider the following linear model for the θ_{ij}:

$$\underline{\theta}_{ab\times 1} = \underline{X}_{ab\times ab}\,\underline{\beta}_{ab\times 1} ,$$

where $\underline{X}$ is the design matrix of the factorial experiment in reduced form so as to be nonsingular. For example, if a=2, b=3,

$$\underline{X} = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} .$$

A model of this type is illustrated by Draper and Smith (1966, p. 257).

Interest centers on the last ab-1 parameters in $\underline{\beta}$, since they represent, respectively, the main effects of A, the main effects of B, and the AB two-factor interactions. Typically, the first parameter in $\underline{\beta}$ is of little interest since it represents the grand mean.

The first step in the estimation of $\underline{\beta}$ is the estimation of $\underline{\theta}$. The results quoted in Section 2 are used to compute $\hat{\theta}_{ij}$ and $\hat{V}(\hat{\theta}_{ij})$, the MLE of θ_{ij} and an estimate of its variance, for each treatment combination (i,j), using the data available on that combination. For each of the models considered in Section 2, $\hat{\theta}_{ij}$ is known to be asymptotically normally distributed with mean θ_{ij} and variance $V(\hat{\theta}_{ij})$. Since independent samples are observed at each treatment combination, $\hat{\underline{\theta}}\sim AN[\underline{\theta},\underline{V}(\hat{\underline{\theta}})]$, where $\underline{V}(\hat{\underline{\theta}})$ is an ab×ab diagonal matrix, whose (k,k) element is $V(\hat{\theta}_k)$, the variance of the k'th element of $\underline{\theta}$. Since $\underline{\beta}=\underline{X}^{-1}\underline{\theta}$, $\hat{\underline{\beta}}$, the MLE of $\underline{\beta}$, is $\hat{\underline{\beta}}=\underline{X}^{-1}\hat{\underline{\theta}}$. Also, $\hat{\underline{\beta}}\sim AN[\underline{\beta},\underline{X}^{-1}\underline{V}(\hat{\underline{\theta}})(\underline{X}^{-1})']$. Letting $\underline{C}=(c_{ij})=\underline{X}^{-1}$, it is easy to see that

$$\frac{\hat{\beta}_k - \beta_k}{\left[\sum_{j=1}^{ab} c_{kj}^2 V(\hat{\theta}_j)\right]^{\frac{1}{2}}} \sim AN(0,1) .$$

Use of this statistic is complicated by the fact that $V(\hat{\theta}_j)$ may depend on the unknown parameter θ_j. However, since $\hat{\theta}_j$ is a strongly consistent estimator for θ_j, $\hat{\theta}_j$ may be substituted for θ_j in the variance, yielding $\hat{V}(\hat{\theta}_j)$, the estimated variance of $\hat{\theta}_j$, and the following result still holds (Cramér, 1945, p. 254):

$$\frac{\hat{\beta}_k - \beta_k}{\left[\sum_{j=1}^{ab} c_{kj}^2 \hat{V}(\hat{\theta}_j)\right]^{\frac{1}{2}}} \sim AN(0,1) .$$

Define

$$V(\hat{\beta}_k) = \sum_{j=1}^{ab} c_{kj}^2 V(\hat{\theta}_j) .$$

The experimenter who wishes to test hypotheses about individual effects now has sufficient tools to proceed. A Bonferroni inequality can be used to develop simultaneous confidence intervals for the β_k's or to develop a test procedure which controls the experimentwise error rate (EER) at some prescribed level α. The EER is defined to be the probability of declaring at least one false positive in the analysis of the experiment, if in fact all effects are equal to zero. Note that β_1, the grand mean, is of no interest here, so that ab-1 β_k's are being tested. The Bonferroni inequality (Miller, 1966, p. 8) states that

$$P\Big(|\hat{\beta}_k-\beta_k|/[V(\hat{\beta}_k)]^{\frac{1}{2}} \le z\{1-\alpha/[2(ab-1)]\},\ \text{for } k = 2,3,\ \ldots,\ ab\Big)$$

$$\ge 1-\sum_{k=2}^{ab} P\Big(|\hat{\beta}_k-\beta_k|/[V(\hat{\beta}_k)]^{\frac{1}{2}} > z\{1-\alpha/[2(ab-1)]\}\Big)$$

$$= 1 - \alpha ,$$

where $z\{\gamma\}$ denotes the 100(1-γ)th percentile of the standard normal distribution. Thus, a procedure yielding EER less than or equal to α is to declare β_k significantly different from zero if

$$|\hat{\beta}_k|/[V(\hat{\beta}_k)^{\frac{1}{2}} > z\{1-\alpha/[2(ab-1)]\},\ \text{for } k=2,3,\ldots,ab.$$

With probability greater than or equal to 1-α , all β_k are contained in the intervals

$$\hat{\beta}_k \pm z\{1-\alpha/[2(ab-1)]\}[V(\hat{\beta}_k)]^{\frac{1}{2}},\ \text{for } k=2,3,\ldots,ab .$$

4. An illustration of the use of the general procedure

This section presents the application of the procedure described in Section 3 to the data from the 2^4 factorial experiment discussed in Section 1. The extensions of the procedure required for the analysis of data from other designs are straightforward.

To begin, an exponential model for the "time to helping" random variable will be tentatively entertained. This model is reasonable, given the structure of the example being considered here.

Let P, N, S, R denote the four factors in the design, each of which appears at two levels. Let

t_{hijkm} = time to helping for trial m at level h of factor P (college proximity), level i of factor N (racial composition of neighborhood), level j of factor S (sex), level k of factor R (race), $m = 1,2, \ldots, M$,

denote M observations of the random variable T_{hijk}, $h,i,j,k=1,2$. In the example considered here "helping" occurs when someone stops to help the confederate and "replication m" is the m'th trial at a particular treatment combination. Assume

$$f(t_{hijkm}) = (1/\theta_{hijk})\exp(-t_{hijkm}/\theta_{ijkm}) ,$$

$$0 \le t_{hijkm} < \infty , \; 0 < \theta_{hijk} < \infty .$$

Now the t_{hijkm} are known exactly only if $t_{hijkm} \le T_0 = 900$ seconds, the censoring time. Define

$$x_{hijkm} = t_{hijkm} , \text{ if } t_{hijkm} \le T_0 ,$$
$$= T_0 , \text{ if } t_{hijkm} > T_0 ,$$
$$r_{hijk} = \text{number of } t_{hijkm} \le T_0 ,$$

= number of uncensored observations on treatment combination hijk, $h,i,j,k, = 1,2$.

Bartholomew (1963) noted that the MLE of θ_{hijk} is

$$\hat{\theta}_{hijk} = \sum_{m=1}^{M} x_{hijkm} \Big/ r_{hijk} ,$$

and that the sampling distribution of $\hat{\theta}_{hijk}$ is asymptotically normal with mean θ_{hijk} and variance

$$V^*(\hat{\theta}_{hijk}) = \theta^2_{hijk}\{M[1 - \exp(-T_0/\theta_{hijk})]\}^{-1} .$$

He also showed that for small M, y_{hijk} (x in his notation) is more nearly normal than $\hat{\theta}_{hijk}$, where

$$y_{hijk} = r_{hijk}(\hat{\theta}_{hijk} - \theta_{hijk}) \Big/ \left(\theta_{hijk}\{M[1 - \exp(-T_0/\theta_{hijk})]\}^{\frac{1}{2}}\right) .$$

The following material assumes that

$\hat{\theta}_{hijk} \mathrel{\dot\sim} N[\theta_{hijk}, V(\hat{\theta}_{hijk})]$, where

$$V(\hat{\theta}_{ijk}) = \theta^2_{hijk}\{M[1 - \exp(-T_0/\theta_{hijk})]\} \Big/ r^2_{hijk} .$$

To express the θ_{hijk} in terms of the usual reduced model for a 2^4 factorial experiment, write

$$\underline{\theta}_{16\times1} = \underline{X}_{16\times16}\, \underline{\beta}_{16\times1} ,$$

where $\underline{X}$ denotes the design matrix for the 2^4 experiment. To illustrate the structure of $\underline{X}$, consider a 2^2 experiment. In this situation

$$\underline{X} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} .$$

For the 2^4 experiment β_1 represents the grand mean; β_2, β_3, β_4, β_5 represent the main effects of the four factors; $\beta_6, \beta_7, \ldots, \beta_{11}$, the two-factor interactions; β_{12}, β_{13}, β_{14}, β_{15}, the three-factor interactions; and β_{16}, the four-factor interaction. Note that $\underline{X}\,\underline{X}' = 16\underline{I}$, where $\underline{I}$ denotes the 16×16 identity matrix. Hence, $\underline{X}^{-1} = (1/16)\underline{X}'$. This implies that $\underline{\beta} = \underline{X}^{-1}\underline{\theta} = (1/16)\underline{X}'\underline{\theta}$ and the MLE of $\underline{\beta}$ is given by $\hat{\underline{\beta}} = (1/16)\underline{X}'\hat{\underline{\theta}}$. Thus, the analysis first uses the individual observations to estimate the parameter θ for each treatment combination. Then the $\hat{\theta}$'s are used to estimate the parameters in the linear model. Individual parameters can be estimated by noting that

$$\hat{\beta}_i = (1/16)\underline{X}_i'\hat{\underline{\theta}} = (1/16)\sum_{j=1}^{16} X_{ij}\hat{\theta}_j ,$$

where $\underline{X}_i'$ denotes the i'th row of $\underline{X}$. Thus,

$\hat{\beta}_i \mathrel{\dot\sim} N[\beta_i, V(\hat{\beta})]$, where

$$V(\hat{\beta}) = (1/16)^2 \sum_{j=1}^{16} V(\hat{\theta}_j) .$$

A Bonferroni-type procedure as described in Section 3 may now be used to test hypotheses about the parameters in the linear model, while controlling the EER. Table 1 gives $\hat{\beta}_i$, $i=2,3,\ldots,16$, and a set of 95% confidence intervals for each of the parameters. These intervals correspond to the usual 0.05 level F-tests of individual effects and interactions performed on the results of a 2^4 factorial experiment using the conventional normal theory model. A second set consisting of 100(1-0.05/15)%=99.67% confidence intervals for each of the parameters can be constructed. The intervals in this second set are of the form $\hat{\beta}_i \pm 546.9$. Using these intervals to test hypotheses about the parameters yields EER≤0.05. An examination of Table 1 indicates that only the main effect of sex is significant when the individual parameters are tested at an 0.05 level. Using EER≤0.05, this effect is almost significant. Hence, this analysis of this experiment indicates that the only factor which significantly affected the mean time to helping was the sex of the confederate. Women were helped much faster than men.

A complete analysis of these data should also include a scrutiny of the assumption in the tentatively entertained model that the times to helping are exponentially distributed. Considerable work has been done on the question of model choice in this situation. Fercho and Ringer (1972) examined four tests of exponentiality and recommended the Gnedenko test as given by Mann, *et al.* (1974) when testing against a Weibull alternative in the presence of censored data. When this test was applied to these data at each of the treatment combinations, the hypothesis that the times are exponentially distributed was not rejected for any treatment combination.

It is exceedingly important to use a model which properly recognizes the censoring and the exponentiality of the data . To demonstrate this, an analysis using a conventional fixed effects analysis of variance model will now be presented. This approach is based on the assumption that the data are normally distributed and not censored. Not surprisingly this conventional model is employed by West *et al.* (1974) in their analysis of these data. The parameter estimates under this model, denoted $\hat{\beta}_1^N$, are given in the last column of Table 1. Comparing $\hat{\beta}_1^N$ to $\hat{\beta}_1$ indicates the dire consequences of ignoring the censoring and exponentiality. Many parameters are seriously underestimated. The normal model also seriously understates the variation in the parameter estimates. For the normal model, 95% and 99.67% confidence intervals are of the form $\hat{\beta}_1^N \pm 53.3$ and $\hat{\beta}_1^N \pm 78.9$, respectively. For the exponential model, they are $\hat{\beta}_1 \pm 367.1$ and $\hat{\beta}_1 \pm 546.9$, respectively. These difficulties stem from the following facts. Under the normal model ignoring the censoring, the treatment combination means are determined by dividing the total time on test (the sum of the eight times to helping) by eight, the number of subjects tested on that treatment combination. However, the correct estimate of the mean, as determined under the exponential model, is evaluated by dividing the total time on test by the number of failures. For instance, for the treatment combination RNP, the estimate of the mean time to helping using the normal model ignoring censoring is $\hat{\theta}^N_{RNP}=5940/8=742.5$ seconds. Using the exponential model which utilizes the information on censoring, the estimate is $\hat{\theta}_{RNP}=5940/3=1980$ seconds.

It should be noted that the purpose of this section has been to illustrate the use of the procedure described in Section 3. Had the purpose been to present an extensive analysis of the data set, additional variables, such as cars per minute and the race of the passing motorists, would have been considered. The complete analysis of these data is considered in another report.

5. Concluding comments

Two cautionary remarks should be made. The method proposed here is based on the asymptotic normality of MLE's. Thus, it should be used with caution when the number of replications at individual treatment combinations is small, as results by Bartholomew (1963) and Billman, Antle, and Bain (1972) indicate. Also, Zelen and Dannemiller (1961) have demonstrated the lack of robustness of procedures based on the exponential model when the true model is Weibull with shape parameter less than one. Hence, as in other applications, in the words of G. E. P. Box, one should not make the Pygmalian mistake of falling in love with the model. If the data indicate that the model is likely to be Weibull, the MLE's for the Weibull parameters should be used.

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TABLE 1 - *Effect estimates under the exponential and normal models and 95% confidence intervals for the effects under the exponential model.*

Effect β_i	$\hat{\beta}_i$	95% Confidence Interval	$\hat{\beta}_i^N$
R	-25.9	(-393.0, 341.2)	26.1
S	-495.3	(-862.4, -128.2)	-108.0
N	29.1	(-337.9, 396.2)	17.9
P	182.4	(-184.7, 549.4)	41.6
RS	94.6	(-272.5, 461.6)	0.5
RN	106.5	(-260.5, 473.6)	8.6
RP	-62.3	(-429.4, 304.7)	-8.6
SN	-51.4	(-418.4, 315.7)	-20.8
SP	-173.3	(-540.4, 193.8)	-11.8
NP	-169.1	(-536.2, 197.9)	-27.9
RSN	-61.5	(-428.6, 305.6)	13.0
RSP	2.4	(-364.7, 369.4)	-10.5
RNP	263.6	(-103.4, 527.3)	54.6
SNP	192.9	(-174.2, 560.0)	31.2
RSNP	-122.4	(-489.5, 244.6)	17.2

$\hat{\beta}_i$ = estimate of β_i under the censored exponential model.

$\hat{\beta}_i^N$ = estimate of β_i under the normal model, ignoring the censoring.